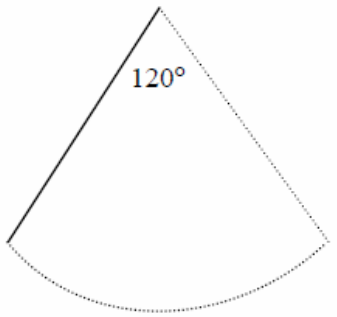
2006 : Moments of Inertia Question

8. **(a)** Prove that the moment of inertia of a uniform rod of mass m and length 2ℓ about an axis through its centre perpendicular to the rod is $\frac{1}{3}m\ell^2$.

(b) A uniform rod of mass $3m$ and length 1.2 metres can turn freely in a vertical plane about a horizontal axis through one end.

The rod oscillates through an angle of $120°$, as shown in the diagram.



(i) Find the angular velocity of the rod when the rod is vertical.

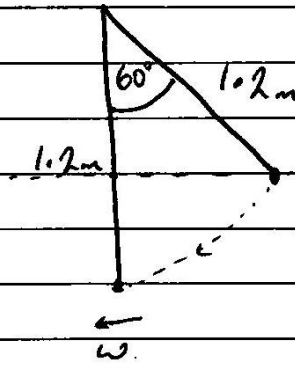
(ii) Find, in terms of m, the vertical thrust on the axis when the rod is vertical.

2006

Q8

(a) PROOF

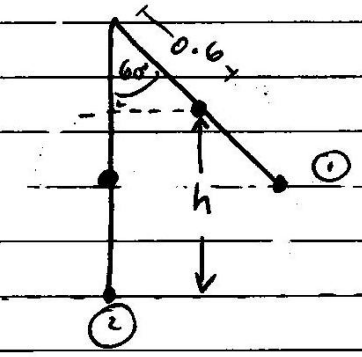
(b)



$I = \frac{4}{3} mL^2$

$I = \frac{4}{3} (3m)(0.6)^2$

$I = 1.44m.$ 5



$h = 1.2 - 0.6 \cos 60°$

$h = 1.2 - 0.6 (\frac{1}{2})$

$h = 1.2 - 0.3$

$h = 0.9m$ 5

ENERGY AT (1): P.E. ROD + K.E. ROD

$mg\bar{h} + \frac{1}{2} I \omega^2$

$(3m)(g)(0.9) + \frac{1}{2}(1.44m)(0)^2$

$2.7mg + 0$

$\boxed{2.7mg}$

ENERGY AT (2): P.E. ROD + K.E. ROD

$mg\bar{h} + \frac{1}{2} I \omega^2$

$(3m)(g)(0.6) + \frac{1}{2}(1.44m)(\omega)^2$

$\boxed{1.8mg + 0.72m\omega^2}$

Energy at ① = Energy at ②

$2.7mg = 1.8mg + 0.72mr\omega^2$ 5

$0.9mg = 0.72mr\omega^2$

$\sqrt{1.25g} = \omega$

$3.5 \text{ rad/sec} = \omega$ 5

(ii) Overall force = Centripetal force (towards centre)

$R - 3mg = K$

$R - 3mg = (3m)r\omega^2$ 5

r = radius of rotation, from axis to centre of gravity $= 0.6m$

$R - 3mg = 3m(0.6)(3.5)^2$

$R = 22.05m + 3mg$

$R = 51.45m$ N. 5